(International) on Combustion, The Williams and Wilkins Co., Baltimore, Md., 1953, pp. 893–906.

[22] Green, L., Jr., "Observations on the Irregular Reaction of Solid Propellant Charges," *Jet Propulsion, American Rocket Society Journal*, Vol. 26, No. 8, Aug. 1956, pp. 655–659.

[23] Price, E. W., "Review of Combustion Instability in Solid Propellant Rocket Motors," TN 5008-4, Feb. 1961, U.S. Naval Ordnance Test Station, China Lake, Calif.

[24] Wimpress, R. N., *Internal Ballistics of Solid-Fuel Rockets*, 1st ed., McGraw-Hill, New York, 1950.

[25] Berl, W. G., "Combustion Instability of Solid Rocket Propellants and Motors," Review Paper 2, July 1962, Solid Propellant Information Agency, The Johns Hopkins Univ., Applied Physics Lab., Silver Spring, Md.

[26] Berl, W. G., Hart, R. W., and McClure, F. T., "Solid Propellant Instability of Combustion, 1964 Status Report," TG 371-8A, July 1964, The Johns Hopkins Univ., Applied Physics Lab., Silver Spring, Md.

[27] Anderson, A. B. C. and Hunt, M. H., "Operational Characteristics of the DynaGage (General Motors Gage) in Investigations of Pressures in Statically Fired Rockets," NOTS 150, Sept. 1948, U.S. Naval Ordnance Test Station, Inyokern, Calif.

[28] Swanson, C. D., "Resonance Burning in Rocket Grains," NOTS TM No. 439, May 1951, U.S. Naval Ordnance Test Station, Inyokern, China Lake, Calif.

[29] Price, E. W., "Axial Mode, Intermediate Frequency Combustion Instability in Solid Propellant Rocket Motors," AIAA Paper 64-146, New York, 1964.

[30] Roberts, A. K. and Brownlee, W. G., "Nonlinear Longitudinal Combustion Instability: The Influence of Propellant Composition," AIAA Paper 69-480, New York, 1969.

[31] Brownlee, W. G., "Nonlinear Axial Combustion Instability in Solid Propellant Motors," *AIAA Journal*, Vol. 2, No. 2, Feb. 1964, pp. 275–284.

[32] Dickinson, L. A., Capener, E. L., and Kier, R. J., "Research on Unstable Combustion in Solid Propellant Rockets," SRI Project PRU-4865, Jan. 1965, Stanford Research Institute, Menlo Park, Calif.

Comments on "Study of Nonlinear Systems"

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A FEW points of the recent comments by Rao,[1] have been advanced in the light of several published materials.

It is a new contribution to apply the direct interchange between dependent and independent variables. Nonlinear, ordinary differential equations, especially Riccati equations, have been actively discussed among radio engineers for nonuniform transmission lines where the line parameters vary continuously along the line length. However, a preliminary literature search seems to exclude the direct interchange between dependent and independent variables.

One of the key points in obtaining closed form solutions by factorizations in Ref. 1 has been the following relationship:

$$(d/dx)[F(x)/f(x)] = \lambda f(x) \tag{1}$$

This is a Bernoulli nonlinear differential equation for $f(x)$ when $F(x)$ and λ are specified,

$$df(x)/dx = f(x)\ (d/dx)[\ln F(x)] - [\lambda/F(x)]f^3(x) \tag{2}$$

This interrelationship between $f(x)$ and $F(x)$ is restricting a possible wider application of Rao's method. In the process of generalizing the original Konyukov's nonlinear differential equation[2,3] a similar relation has been derived for variable coefficients.[4] It is also noted that the book by Murphy[5] goes into a detailed discussion on Abel's nonlinear differential equations, including the separable case of Eq. (1).

At this point, Rao's method has been used for a generalized Konyukov's nonlinear differential equation. The direct interchange of dependent and independent variables has been given to

$$x d^2x/dt^2 + P(t)(dx/dt)^2 + Q(t)x^3 + R(t)x^2 = 0 \tag{3}$$

where $P(t)$, $Q(t)$, and $R(t)$ are arbitrary functions of the independent variable t. Then, the result is

$$d^2t/dx^2 = [P(t)/x]\, dt/dx + [Q(t)x^2 + R(t)x](dt/dx)^3 \tag{4}$$

Evidently this appears not solvable for general $P(t)$, $Q(t)$, and $R(t)$. In contrast, the published materials show how to obtain closed forms for the following cases:

$P(t) = -2$, $Q(t) =$ constant, $R(t) =$ constant, Ref. 2

$P(t) = -2$, $Q(t) =$ arbitrary, $R(t) =$ constant, Ref. 3

$P(t) = -2$, $Q(t) =$ arbitrary, $R(t) =$ a solution of Bernoulli's equation Ref. 4

Equation (4) is solvable if $P(t)$, $Q(t)$, and $R(t)$ are all constant, since it then becomes a Bernoulli's nonlinear differential equation for dt/dx. Thus, it appears that another key point of why Rao's method works successfully on Eq. (2) of Ref. 1 is that it contains no variable coefficients of t.

References

[1] Rao, M. N., "Comments on 'Study of Nonlinear Systems'," *AIAA Journal*, Vol. 8, No. 6, June 1970, pp. 1183–1184.

[2] Konyukov, M. V., "Nonlinear Langmuir Electron Oscillations in a Plasma," *Journal of Experimental and Theoretical Physics of the Academy of Sciences of the USSR* (English translation), Vol. 37, No. 10, March 1960, pp. 570–571.

[3] Sugai, I., "Exact Solutions for Ordinary Nonlinear Differential Equations," *Electrical Communications*, Vol. 37, No. 1, May 1961, pp. 47–55.

[4] Sugai, I., "Changes of Variables for Generalized Konyukov Second-Order Nonlinear Differential Equations," *Proceedings of the Institute of Electrical and Electronics Engineers (Correspondence)*, Vol. 53, No. 12, Dec. 1965, p. 216.

[5] Murphy, G. M., *Ordinary Differential Equations and Their Solutions*, D. Van Nostrand, Princeton, N. J., 1960, pp. 23–26.

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Comment on "Unsymmetrical Bending of Shells of Revolution"

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IN Ref. 1, Blech has presented a set of eight first-order ordinary differential equations for the bending analysis of unsymmetrically loaded shells of revolution on the basis of Sander's first-order shell theory. This formulation was first suggested by Goldberg et al.[2] and has been successfully employed by several authors for the equilibrium, stability and vibration problems of shells.

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